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Application of Combinatorial Approach in Packaging Material Selection

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ABSTRACT

Packaging material selection (PMS) problems have always been important to packaging designers and engineers. Not only does the selection of packaging material determine the costs and the environmental impacts of packaging, but also influences packaging physical characteristics and associated manufacturing methods. In order to reduce economic and environmental impacts, one has to take a holistic approach to packaging material selection by considering material effects throughout the packaging life cycle. To evaluate economic costs and environmental impacts both quantitative factors and subjective criteria play an important role in the packaging design. In the present work, fuzzy set theory is used for representing and manipulating the vague and subjective descriptions of packaging performance and design attributes. Further a genetic algorithm based approach is used for addressing the packaging material selection problem through multiple criteria decision-making. The overall approach comprises of two phases. In the first phase, fuzzy set theory is used for the linguistic transformation of performance attributes into numerical values. It results in a decision matrix that contains crisp scores. Also in this phase, a weight is assigned to each sub-criterion to show its importance compared to others. In the second phase, a GA is used to globally search for near-optimal or optimal design solutions. The implementation of the proposed methodology is illustrated through a numerical example.

Keywords: Packaging, Material Selection, Multiple Criteria Decision-Making, Genetic Algorithms, Fuzzy Set Theory.

1. INTRODUCTION AND REVIEW OF LITERATURE

Packaging material selection (PMS) problems have always been important to packaging designers and engineers. The decision regarding packaging materials determine not only the costs and environmental impacts of packaging, but also the physical characteristics, associated manufacturing processes and product recovery. Usually, economic and environmental considerations dominate the packaging design decisions. To strike a balance between economic and environmental factors, it is necessary to take a holistic decision-making approach like “life cycle engineering or design”. In general, life cycle engineering centers on the design and production that offers minimal costs and environmental impacts during entire life cycle of the products. Within this methodology, an innovative technique such as life cycle analysis is used to provide decision-makers, whether consumers, industrialists, or government policy makers, with information that will allow them to understand the environmental impacts of their actions. Life cycle analysis supports decisions related to material selection, design optimization, and process selection.

Many researchers have used the LCA approach to packaging and containers. The group of research include Keoleian and Spitzley, Van Doorselaer and Fox, Oki and Sasaki, Freire et al, Hekkert et al, Barbiroli and Raggi, Pun et al, Ross and Evans, Zabaniotou and Kassidi, and Khan et al.

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In addition to quantitative measures of economic costs and environmental impacts, subjective criteria like packaging performances also play an important role in the packaging design decision-making. A good design aims at creating packaging that meets expected performance levels which may be represented in terms of quality, functionality, appearance, manufacturability, and so on. Packaging performance is normally characterized in terms of qualitative verbal descriptions. Fuzzy set theory has provided a powerful approach to represent and manipulate such non-quantitative descriptions through different operations of fuzzy sets and fuzzy numbers. For example, linguistic expression such as “easy to assemble” and “high manufacturability” can be captured by appropriate fuzzy numbers and finally defined by crisp (real) numbers.

In fuzzy sets, the linguistic expressions are transformed to triangular fuzzy numbers (TFN). TFN consists of parameters $a$, $b$, and $c$ (Figure 1). Each parameter represents a quantity of a linguistic value associated with a degree of membership in the interval [0, 1]. Further, these three parameters denote the smallest possible quantity, the most promising quantity, and the largest possible quantity that describe the linguistic value.

![Figure 1: Triangular fuzzy number (TFN)](image)

In the past few years, many researchers have applied fuzzy principles in product design that has encountered the great amount of fuzzy information. Zhou, for example, captured problem characteristics by employing fuzzy set approach to solve the problem to improve product design and quality. Bohez et al. combined both fuzzy sets and analytical hierarchy process (AHP) for making good choices from a number of feasible alternatives of flexible manufacturing system. Later, Tsai and Hsiao developed fuzzy-based principles for searching through possible alternative combinations for multi-functional product customization. Other researchers have applied fuzzy set theory for evaluating designs that need to satisfy multiple criteria. In evaluating design alternatives, attributes that express the desirability of candidate design solutions are expressed as fuzzy numbers. For instance, Hsiao developed a fuzzy decision-making method for selecting an optimal design. In his work, the evaluation objectives are arranged in a multi-level hierarchical structure and then each objective is assigned a function that determine its contribution to the overall rating of the design solution. This approach is known as a fuzzy-weighted average (FWA) method. Later, Vanegas and Labib prescribed the use of fuzzy numbers and extended the FWA method to facilitate the calculations to produce less imprecision but more credible and realistic results. Recently, Gupta et al. presented a fuzzy impact analysis technique for the selection of landfill sites.

There are many techniques that address design problems involving multiple criteria evaluation. Among them, the genetic algorithm-based approaches are successfully applied to solve complex optimization problems. Genetic algorithm (GA) was first proposed and studied by John Holland. It is a probabilistic approach that relies on randomization techniques. GA has received considerable attention for its potential in optimization technique for complex problems. Though there are many variations of GAs, they all follow the same basic principles of natural genetics and natural selection for finding good or near-optimal solutions to various combinatorial problems. GAs search through a solution space without preconceptions about what is possible and what is not. GAs conduct solution search from one population to another, rather than from one individual solution to another individual solution. They use objective information rather than derivative to guide through the solution space. As a result, the only information they need is a measure of fitness in many different areas of the solution space at once. This gives GAs the power to search noisy spaces littered with local optimum.
Many researchers have applied GAs for material design problems. For example, Haidar et al. [9] applied GAs for the equipment selection and testing in opencast mining problems. Yoshimura and Izui [28] used GAs for solving global optimization problem from both design and manufacturing point of view by constructing hierarchically structured decision variables. Kamrani and Gonzalez [13] have demonstrated the application of GA for finding a solution to a modular design problems. Yang et al. [27] proposed an automatic procedure to select the optimal material and manufacturing conditions for composite materials. Their procedure uses multiple neural networks to represent the most important characteristics of the composite material, and GA assigns to select the optimal processing conditions.

In the present work, a GA-based approach is taken to solve a package material selection problem, which involves a multiple criteria decision-making. The approach comprises of two phases. In the first phase, fuzzy set theory is used for transforming linguistic expressions into numerical (crisp) values. The result of the first phase is decision matrix that contains crisp scores. Also in this phase, a weight is assigned to each sub-criterion to show its importance compared to others. In the second phase, a GA is used to globally search for optimal or near-optimal design solutions.

This paper has the following sections. Section 2 gives a brief theoretical background for fuzzy transformation and genetic algorithm approaches. Section 3 identifies economical, environmental and robustness criteria that will be used in the PMS problem. Then, the solution methodology is described in Section 4. A numerical example is presented in Section 5. Finally, the conclusions are stated in Section 6.

2. THEORETICAL BACKGROUND

Fuzzy transformation using fuzzy set theory and the genetic algorithm are described in the following.

2.1 Fuzzy Transformation

Each TFN has linear representations on its left and right side such that the membership function \( \mu(x) \) can be represented as:

\[
\mu(x) = \begin{cases} 
0, & x < a \\
\frac{x-a}{b-a}, & a \leq x \leq b \\
\frac{x-c}{b-c}, & b \leq x \leq c \\
0, & x > c 
\end{cases}
\]

\( \forall x \in X, \mu(x) \in [0, 1] \) (1)

A quantity \( x \) of the linguistic value is defined over the universe of quantified linguistic values \( X \), which is essentially a set of real numbers. When \( x \) increases from \( a \) to \( b \), its corresponding degree of membership linearly increases from 0 to 1. While \( x \) increases from \( b \) to \( c \), its corresponding degree of membership linearly decreases from 1 to 0.

It is possible to experience two or more fuzzy numbers for real-life situation; it is difficult to distinguish which fuzzy numbers are better and which ones aren’t. To deal with a situation like this, there are many methods that convert fuzzy numbers into the crisp value. Such methods include Center-of-Area method, Center-of-Maxima method, and Mean-of-Maxima method. However, in this work, a systematic Left-Right scoring approach proposed by Chen and Hwang [4] is used for converting fuzzy numbers to crisp values.

Consider an example representing the left and right values of fuzzy number \( M \) as shown in Figure 2.
Given a maximizing set and a minimizing set of fuzzy number \( M \) as

\[
\mu_{\text{max}}(x) = \begin{cases} 
  x, & 0 \leq x \leq 1 \\
  0, & \text{otherwise}
\end{cases} 
\]  

(2)

\[
\mu_{\text{min}}(x) = \begin{cases} 
  1-x, & 0 \leq x \leq 1 \\
  0, & \text{otherwise}
\end{cases} 
\]  

(3)

the right, left and total values of \( M \) can be calculated using

\[
\mu_R(M) = \sup_x [\mu_M(x) \wedge \mu_{\text{max}}(x)] 
\]  

(4)

\[
\mu_L(M) = \sup_x [\mu_M(x) \wedge \mu_{\text{min}}(x)] 
\]  

(5)

\[
\mu_T(M) = \frac{\mu_R(M) + 1 - \mu_L(M)}{2} 
\]  

(6)

where \( M \) is a continuous, convex, and normal fuzzy number; \( \mu_R(M) \) can be determined by taking the intersection of the non-increasing part of \( \mu_M(x) \) and \( \mu_{\text{max}}(x) \); and \( \mu_L(M) \) is the intersection of the non-decreasing part of \( \mu_M(x) \) and \( \mu_{\text{min}}(x) \). The crisp value \( r \) is defined as:

\[
r = \mu_T(M) 
\]  

(7)

2.2 Genetic Algorithm (GA)

The basic principle starts with an initial population, which is a set of random feasible solutions that are encoded into character strings or bit strings called “chromosomes.” The unit of chromosome is called “gene.” The chromosomes length depends upon the required precision. Each chromosome is evaluated by using some fitness function related to the objective function value of the optimization problem. Chromosomes with highest fitness values are given a higher probability for being selected as parents. There are many methods for selecting parents (such as roulette wheel selection, Boltzmann selection, rank selection, steady-state selection, etc.). However, the steady-state selection method is used in this work. The steady-state selection GA works in the following way. In every generation a few good (with higher
fitness) chromosomes are selected for creating new offspring. Then some bad (with lower fitness) chromosomes are removed and the new offspring is placed in their place. The rest of population survives to new generation[19].

In order to reproduce offspring for the next generation, the genetic operators are performed. There are two classical genetic operators in GA: crossover and mutation. The crossover operator generates the offspring from parents by swapping their genes at one or more randomly chosen “loci” of the chromosomes. However, the crossover rate should be high—about 80%-95% in general. The crossover rate reflects the likelihood that future scenarios or “organisms” will contain a mix of information from the previous generation of parent organisms. For example, the rate of 60% means that roughly 60% of an offspring organism’s values will come from the first parent and 40% will come from the second parent. A crossover rate of 100% means that no crossover will occur, so only clones of the parents will be evaluated.

The mutation operator arbitrarily changes one or more genes of the selected chromosome to increase the structural variability of the population. The changes are randomly selected by mutation rate. The mutation rate reflects the likelihood that future scenarios will contain some random values. The mutation rate usually falls between 1%-5%. If the mutation rate is too low, many genes that would have been useful are never tried out. If it is too high, it means that more mutations or random gene values will be introduced into the population, thus the offspring will start losing their resemblance to the parents, and the algorithm will lose the ability to learn from the history of the search. Further, chromosomes with lower fitness values are replaced by the offspring. Reproduction repeats until termination condition is met. The termination condition is based on the number of generations in general for most GA applications.

This completes the theoretical review of fuzzy transformation and genetic algorithm. In the next section, the evaluation criteria for material decision-making used in this work are described.

### 3. EVALUATION CRITERIA OF PMS

During the design stage of returnable packaging, a group of packaging materials will be chosen and evaluated to satisfy the certain decision criteria for entire packaging life cycle—including packaging material production, transportation, production process, distribution, use/reuse, and disposal at the end of life cycle (Figure 3).

![Figure 3: Life cycle boundary of returnable packaging](image-url)
There are three types of decision criteria used in this work: (1) economical criteria that relate to cost issues, (2) environmental impacts relate to environmental damages caused by packaging, and (3) packaging robustness that affects the usage rate of packaging. However, the first two types of the decision criteria are measured numerically while the last is qualitatively measured. The descriptions of these three decision criteria are detailed in the following.

3.1 Economical Criteria
Economical criteria are a major product design driver that forms an important share of the total cost. For returnable packaging, the total cost considered can be estimated through an activity-based approach. Certainly, the cost list can be different and may be extended by other enterprise-specific expenses. However in this work, possible costs are grouped into three categories.

**Fixed cost** (FIX): The initial capital or fixed cost includes material cost, cost for machines, appliances and tools, and energy used for manufacturing and/or fabricating of packaging. This can be defined based on unit production yearly or daily and dependent on the marketing requirements.

**Maintenance cost** (MTE): Returnable packaging entails a different set of costs associated with the return of packages and containers to their point of origin, including administrative cost for managing and controlling the flow of full and empty containers, the cost of labor needed to put empty containers back, the freight cost of hauling back empty containers, and the cost of cleaning, repairing, sorting and storing containers.

**End-of-life cost** (EOL): This is the cost caused by deposition and incineration at of packaging at their end-of-life. This includes waste disposal cost and landfill cost.

3.2 Environmental Impacts
These criteria suggest various elements due to environmental impacts (or damages) during life cycle of packaging. At a technical level, a number of environmentally relevant technical aspects for the various stages of the packaging life cycle are quantified and computed. In this work, a list of the selected aspects considered is as follows.

**Resource and energy consumption** (REC): This refers to the efficiency of the transformation-utilization of common raw and auxiliary materials and energy in the form of electricity, power, heat, gas, water and so on introduced into the cycle. This can be measured by calculating the percentage ratio of the quantity of the materials and energy incorporated into packaging to the quantity entered into the cycle.

**Solid waste emission** (SWE): This includes the quantity of solid wastes generated from life cycle and in landfills. This can be measured by calculating the percentage ratio of the amount of solid wastes released per packaging unit.

**Atmospheric emission** (ATE): This is the atmospheric emission caused by the production and energy cycle. This can be expressed as the overall quantity of substances emitted into the atmosphere per packaging unit during the production, energy conversion and use cycle. The substances include particulate solids, chemical sprays, greenhouse gases, methane, carbon monoxide, carbon dioxide and so on.

**Liquid pollutant emission** (LPE): This measures the liquid emission intensity. This is expressed as the quantity of substances, per packaging unit, released into wastewater. The pollutant substances include total suspended solids, biochemical oxygen demand, and chlorinated organic compounds.

**Human toxicity risk** (HTR): This element relates to toxicity of pollutants in terms of cancer and non-cancer risks. Cancer risk is reported in terms of unit risk values whereas non-cancer risks are shown by hazard quotients. The hazard quotients are added for all pollutants and all possible routes (such as ingestion and inhalation). However, this may be expressed by means of concise indices obtained by weighting the various substances according to their degree of human toxicity.

3.3 Packaging Robustness
Not only packaging engineers and designers are responsible for the selection of size and materials, but they also must ensure that the prospective packaging performance meets many requirements as desired. Robustness criteria include packaging functionality, durability, and maintainability.
**Functionality** (FTL): It is a general fact that, almost all goods, agricultural and manufactured pass through similar cycle in their distribution. Immediately after packing, they are warehoused; transported to a distribution center or customer warehouse; moved to a retail outlet; removed from the shippers; displayed as individual packages; and finally, sold and carried to their final destination by purchasers. During this cycle, the products need two kind of protection: climatic and mechanical\[^{[17]}\]. Climatic protection is packaging property that resists damage to the product from atmospheric effects such as temperature, light, humidity, and gas reactions. Mechanical protection is resistance to physical forces, such as impact, abrasion, vibration, and torsion.

**Durability** (DBT): The useful life period of returnable packaging is important for the net present value calculation; the longer the life, the more profitable the investment. The use life depends on the strength of the packaging material and the construction. For instance, steel and plastic have a longer life than wooden packages. There is little reliable information available concerning package life, though there are some tests that can be helpful to generate this data. The technical characteristics of packages, determined by corresponding standards and technical requirements, are verified by durability. Packaging “durability” is defined as “an ability to protect the quality of the product in a package to a desired level and given conditions for a specified period of time.” Concisely, packaging durability must be in terms of the probability of surviving the specified life with satisfactory performance throughout. Normally, the packaging durability is stated in terms of one of the following—probability density function, failure rate, or mean lifetime\[^{[16]}\].

**Maintainability** (MTT): By definition, maintainability is a characteristic of a system expressed as the probability that an item will be retained in (or restored to) a specified condition within a given period of time, when the maintenance is performed in accordance with prescribed procedures and resources. As in returnable packaging systems, packaging designers and engineers concern ergonomics as a relevant factor in packaging maintainability. Package opening, emptying, and handling are designed to improve workers’ productivity. Some returnable packages can be knocked down (or collapsed) or nested to reduce return transport costs, and usually has better cube utilization than expendable packages because they are stronger and more stackable. Further, returnable packaging should be designed for less maintenance, higher manufacturability, or ease of assembly and disassembly for replacement when returnable packaging is fractured, so that it can return back to the distribution system for making used many times.

Figure 4 shows a decision tree on the selection of packaging materials.

### 4. ILLUSTRATION OF SOLUTION APPROACH FOR THE PMS

In this section, the elements and solution methodology with the help of an example as it pertains to the selection problem of packaging materials will be described.

The given approach is composed of two phases. The result of the first phase is decision matrix that contains only crisp values. If there exists qualitative sub-criteria that are expressed through linguistic terms, it is necessary to transform those linguistic terms into fuzzy number first, and then all fuzzy numbers are assigned crisp scores. Also, in this phase, a weight is assigned to each sub-criterion to show its importance by being expressed through linguistic terms. In the second phase, GA is used for globally searching for an optimal or near optimal solution. Each phase will be detailed in the following.

#### 4.1 Scoring Evaluation Criteria by Fuzzy Set Theory

A numerical approximation method is proposed to systematically convert linguistic terms to their corresponding fuzzy numbers (see Figure 5). Further, the left-right scoring method is used for calculating left, right and total values of fuzzy numbers. Finally, the crisp values of fuzzy numbers are calculated and shown in Table 1.
Figure 4: Decision tree of the PMS

Figure 5: Linguistic terms and the corresponding left and right values
Table 1: Linguistic terms and corresponding fuzzy numbers

<table>
<thead>
<tr>
<th>Linguistic Terms</th>
<th>Very Low (VL)</th>
<th>Low (L)</th>
<th>Fairly Low (FL)</th>
<th>Fairly High (FH)</th>
<th>High (H)</th>
<th>Very High (VH)</th>
</tr>
</thead>
<tbody>
<tr>
<td>TFN</td>
<td>(0, 0, 0.2)</td>
<td>(0, 0.2, 0.4)</td>
<td>(0.2, 0.4, 0.6)</td>
<td>(0.4, 0.6, 0.8)</td>
<td>(0.6, 0.8, 1)</td>
<td>(0.8, 1, 1)</td>
</tr>
</tbody>
</table>

\[ \mu_T(M) = r = \begin{bmatrix} 0.0833 \\ 0.2500 \\ 0.4167 \\ 0.5833 \\ 0.7500 \\ 0.9170 \end{bmatrix} \]

For quantitative criteria \((QT)\), it is necessary to normalize the associated values \((v_{i,QT})\) to obtain the scores \((s_{i,QT})\) in the same range as the qualitative \((QL)\) variables, i.e., in the range \([0, 1]\). Equation (8) shows the normalization formula:

\[
s_{i,QT} = \frac{\max_{i=1}^{N}(v_{i,QT}) - v_{i,QT}}{\max_{i=1}^{N}(v_{i,QT}) - \min_{i=1}^{N}(v_{i,QT})}, \text{ for every } QT
\]  

(8)

Equation (8) converts a “lower” value \(v_{i,QT}\) to the “higher” score \(s_{i,QT}\) and vice versa. This reflects that fact that a packaging material that minimizes economical and environmental impacts is desirable.

The linguistic terms are used to indicate the importance of each sub-criterion. The linguistic descriptions are converted to numerical values that weigh \((w_j)\) each sub-criterion \(j\). Because weights range from 0 to 1, the aggregated ranking value \((\bar{r}_j)\) thus falls in the space \([0, 1]\) and can be defined as:

\[
\bar{r}_j = \frac{\sum_{j \in QT,QL} w_j s_{ij}}{\sum_{j \in QT,QL} w_j}
\]  

(9)

4.2 Searching Solution by GA

In this section, an example is provided for better understanding the elements and development of GA of the PMS problem.

Chromosome Structure

As in every GA problem, the search for solution starts by defining solution encoding to chromosomes. All ten packaging materials are encoded into numerical strings. Returnable packaging system is made of ten components. In other words, the number of problem variables equals ten, therefore the chromosome length also equals to ten \((chl = 10)\). An example of the chromosome can be structured as “3, 4, 6, 3, 5, 10, 8, 7, 1, 2”, which is interpreted as “component 1 is made of material type 3; component 2 is made of material type 4, and so on.”

Initial Population

The initial population \((pop = 0)\) consists of \(nchr\) chromosomes. However, if there is some information about the distribution of the optima, such information can be used in generating the initial population.

Evaluation and Parent Selection

Each chromosome \(chr\) is evaluated using a fitness function. In this work, the objective of the evaluation function is to maximize the total ranking value \((\bar{R})\) obtained by adding together aggregated ranking values \((\bar{r}_j)\) of all selected packaging material choices \(k\) in chromosome position \(chr\). Hence,
With $n_{chr} = 9$ in this example, the list of possible chromosomes and fitness values ($FITNS(chr, gnt)$) are evaluated as shown in Table 2. Next, each chromosome’s fitness value is ranked in descending order. Chromosomes with highest fitness values are given a higher probability for being selected as “parents ($P(gnt)$)” for creating a new “offspring ($O(gnt)$).” Some lower-fitness chromosomes are removed and the new offspring is placed in their place.

**Genetic Operations**

In order to improve the initial population, certain string manipulations are necessary. This is done by using genetic operators for every genetic algorithm application, namely, *crossover* and *mutation*. These two genetic operators are explained in the following paragraphs.

The crossover operator generates the offspring from parents by swapping their genes at one or more randomly chosen “loci” of the chromosomes. Crossover operates on selected genes from parent chromosomes to create a new offspring. For example, a new offspring can be created by randomly choosing a crossover point and copying everything before this point from the first parent and then copying everything after the crossover point from the other parent. However, the crossover rate should be high—about 80%-95% in general. As appeared in literature, some results show that for some problems crossover rate about 60% is the best.

Crossover can be quite complicated and depends mainly on the encoding of chromosomes. Specific crossover made for a specific problem can improve performance of the genetic algorithm. The crossover methods include:

- **Single point crossover** (one crossover point is selected): string from the beginning of the chromosome to the crossover point is copied from the first parent, and the rest is copied from the other parent.
- **Two point crossover** (two crossover points are selected) string from the beginning of the chromosome to the first crossover point is copied from the first parent, the part from the first to the second crossover point is copied from the other parent and the rest is copied from the first parent again.
- **Uniform crossover**: genes are randomly copied from the first or from the second parent.
- **Arithmetic crossover**: some arithmetic operation is performed to make a new offspring.

After a crossover is performed, mutation takes place. The mutation operator arbitrarily changes one or more genes of the selected chromosome to increase the structural variability of the population. In the present work this is limited to the range of available packaging materials in the database. The changes are randomly selected by mutation rate. The mutation rate usually falls between 1%-5%. If the mutation rate is too low, many genes that would have been useful are never tried out, and if it is too high, the offspring will start losing their resemblance to the parents, and the algorithm will lose the ability to learn from the history of the search.

**Table 2: A sample of chromosomes for initial population**

<table>
<thead>
<tr>
<th>chr</th>
<th>Chromosome structure</th>
<th>Fitness ($FITNS(chr, gnt)$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2, 3, 4, 6, 8, 8, 10, 5, 2, 4</td>
<td>4.6933</td>
</tr>
<tr>
<td>2</td>
<td>1, 2, 8, 7, 6, 5, 4, 3, 2, 6</td>
<td>5.0769</td>
</tr>
<tr>
<td>3</td>
<td>9, 8, 7, 6, 4, 2, 7, 10, 2, 1</td>
<td>4.8065</td>
</tr>
<tr>
<td>4</td>
<td>3, 4, 5, 6, 1, 3, 9, 4, 8, 10</td>
<td>5.3598</td>
</tr>
<tr>
<td>5</td>
<td>9, 2, 7, 6, 4, 9, 10, 2, 1, 1</td>
<td>5.0925</td>
</tr>
<tr>
<td>6</td>
<td>3, 1, 1, 2, 2, 5, 6, 4, 8, 7</td>
<td>4.9861</td>
</tr>
<tr>
<td>7</td>
<td>10, 9, 7, 6, 3, 1, 2, 10, 8, 7</td>
<td>4.8515</td>
</tr>
<tr>
<td>8</td>
<td>2, 4, 6, 8, 10, 1, 2, 4, 6, 8</td>
<td>5.3514</td>
</tr>
<tr>
<td>9</td>
<td>6, 3, 4, 7, 9, 10, 10, 3, 8, 5</td>
<td>5.0601</td>
</tr>
</tbody>
</table>
As an example consider the crossover and mutation operations given in Tables 3 and 4 respectively.

### Table 3: Crossover operation

<table>
<thead>
<tr>
<th>chr</th>
<th>Before crossover</th>
<th>After crossover</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>3, 4, 5, 6</td>
<td>3, 4, 5, 6, 10, 1, 2, 4, 6, 8</td>
</tr>
<tr>
<td>8</td>
<td>2, 4, 6, 8</td>
<td>2, 4, 6, 8, 1, 3, 9, 4, 8, 10</td>
</tr>
</tbody>
</table>

(Note: | is the crossover point)

### Table 4: Mutation operation

<table>
<thead>
<tr>
<th>Before mutation</th>
<th>After mutation</th>
</tr>
</thead>
<tbody>
<tr>
<td>(3), 4, 5, 6, (1), 3, 9, 4, 8, 10</td>
<td>(5), 4, 5, 6, (3), 3, 9, 4, 8, 10</td>
</tr>
<tr>
<td>2, (4), 6, 8, 10, 1, (2), 4, 6, 8</td>
<td>2, (2), 6, 8, 10, 1, (8), 4, 6, 8</td>
</tr>
</tbody>
</table>

(Note: ( ) is the mutation point)

### Population Replacement

The idea of the *elitism* is introduced. When creating a new population by crossover and mutation, there is a big chance that the best chromosome might be lost. Elitism is the name of the method that first copies the best chromosome (or few best chromosomes) to the new population. The rest of the population is constructed in ways described above. Elitism can rapidly increase the performance of GA, because it prevents a loss of the best found solution. Next, chromosomes with lower fitness values are replaced by the offspring to form a new population.

### Termination Condition

Reproduction repeats until one of the two following termination conditions is met: (1) the number of generations reaches up to a designated value, or (2) the ration of the average fitness value of the new population to the average fitness value of the old population has a value less than or equal to a certain number, meaning, the population is not improving any further. In general, the termination condition is based on the number of generations for most GA applications.

### 4.3 Steps to Solve the PMS Problem

The steps of the proposed approach can be given as follows.

**Step 1.** For each packaging material \( i \), determine the values \( v_{i,QT} \) for quantitative criteria \( QT \) and evaluate qualitative criteria \( QL \) by using linguistic variables. Also, assign weighting \( w_j \) to each criterion \( j \).

**Step 2.** Normalize \( v_{i,QT} \) to obtain the associated score \( s_{i,QT} \) in the range \([0, 1]\) by using Equation (8). Transform linguistic variables to numerical (crisp) values.

**Step 3.** Calculate ranking values \( \bar{r}_i \) of each packaging material by using Equation (9).

**Step 4.** Start the GA search with a randomly selected initial population, which consists of \( nchr \) chromosomes with chromosome length \( chrl \).

**Step 5.** Calculate the fitness value \( FITNS(chr, gnt) \) of each chromosome \( chr \) in the generation \( gnt \) by Equation (10).

**Step 6.** Sort the chromosomes in a descending order according to their fitness.

**Step 7.** Select two parent chromosomes \( (P_1 \) and \( P_2) \) based on the fitness bias.

**Step 8.** Randomly calculate the crossover probability. If the probability holds, generate the output of the crossover operation as the offspring. If not, define parents \( (P_1 \) and \( P_2) \) as the offspring.

**Step 9.** Place the offspring into a new generation \( (gnt+1) \). Return to Step 2.

**Step 10.** Randomly calculate the mutation probability. If the probability holds, perform the mutation operation to generate the output of the mutation as new generation. If not, define the current population as new population.

**Step 11.** Replace the offspring \( (O_1 \) and \( O_2) \) in the new generation pool \( (gnt+1) \).
Step 12. Use new generated population for a further run of the algorithm.  
Step 13. Stop the algorithm if the termination condition is satisfied. Otherwise, return to Step 5.

5. NUMERICAL EXAMPLE

Consider returnable packaging whose structure comprises of nine components \((K = 9)\). Each component is indexed by integers ranging from 1 to 9, i.e. \(\{k \in K / k = 1, 2, \ldots, K\}\). For each component \(k\), one of ten packaging material types \((N = 10)\) is selected, i.e. \(\{i \in n / i = 1, 2, \ldots, N\}\). There are eleven evaluation sub-criteria that are used for selecting material choices for each component. These sub-criteria are divided into two groups: quantitative \((QT)\) and qualitative \((QL)\) sub-criteria. That is,

\[
QT \in \{\text{FIX}, \text{MTE}, \text{EOL}, \text{REC}, \text{SWE}, \text{ATE}, \text{LPE}, \text{HTR}\}
\]

\[
QL \in \{\text{FTL}, \text{DBT}, \text{MIT}\}
\]

Table 5 shows the sample data of quantitative and linguistic values obtained by all packaging materials for component 1 through the evaluation sub-criteria. Table 5.6 shows the numerical results of quantitative and qualitative criteria as well as the aggregated ranking value of each packaging material for component 1. As shown in Tables 5 and 6, the value of \(v_{i=6,\text{FIX}} = $4,187\) will receive the score \(s_{i=6,\text{FIX}} = 0\); while \(v_{i=4,\text{FIX}} = $547\) receives the score \(s_{i=4,\text{FIX}} = 1.0000\) after the normalization. Also, Table 7 summarized the aggregated ranking values of packaging material \(i\) for each component.

<table>
<thead>
<tr>
<th>Sub-criteria ((j))</th>
<th>Degree of importance</th>
<th>Packaging material ((i))</th>
</tr>
</thead>
<tbody>
<tr>
<td>(QT)</td>
<td></td>
<td>1</td>
</tr>
<tr>
<td>FIX</td>
<td>H</td>
<td>1254</td>
</tr>
<tr>
<td>MTE</td>
<td>H</td>
<td>160</td>
</tr>
<tr>
<td>EOL</td>
<td>FH</td>
<td>50</td>
</tr>
<tr>
<td>REC</td>
<td>FL</td>
<td>99</td>
</tr>
<tr>
<td>SWE</td>
<td>L</td>
<td>63</td>
</tr>
<tr>
<td>ATE</td>
<td>L</td>
<td>51</td>
</tr>
<tr>
<td>LPE</td>
<td>FL</td>
<td>21</td>
</tr>
<tr>
<td>HTR</td>
<td>FL</td>
<td>97</td>
</tr>
<tr>
<td>(QL)</td>
<td></td>
<td>218</td>
</tr>
</tbody>
</table>

Table 5: Evaluation matrix of packaging material \(i\) for component 1
Table 6: Normalized scores of packaging material \( i \) for component 1

<table>
<thead>
<tr>
<th>Sub-criteria ((j))</th>
<th>Weight ((w_j))</th>
<th>Packaging material ( i )</th>
</tr>
</thead>
<tbody>
<tr>
<td>FIX</td>
<td>0.7500</td>
<td>0.8058 0.9717 0.9926 1.0000 0.8973 0.6338 0.5418 0.8283 0.9571</td>
</tr>
<tr>
<td>MTE</td>
<td>0.7500</td>
<td>0.8965 0.8966 0.7905 0 0.1724 0.8793 0.8060 1.0000 0.9957 0.7802</td>
</tr>
<tr>
<td>EOL</td>
<td>0.5833</td>
<td>0.5600 0.7600 0.6400 0.6400 0.0400 1.0000 0 0.3200 0.2400 0</td>
</tr>
<tr>
<td>REC</td>
<td>0.4167</td>
<td>0.0167 0.0500 0.1333 0.4333 0.7167 1.0000 0 0.1167 0.1833 0.4833</td>
</tr>
<tr>
<td>SWE</td>
<td>0.2500</td>
<td>0.4253 1.0000 0.4483 0.6552 0 0.0115 0.5977 0.1839 0.6667 0.0115</td>
</tr>
<tr>
<td>ATE</td>
<td>0.2500</td>
<td>0.6071 0.4107 1.0000 0.9643 0.7321 0.7143 0 0.4107 0.5000 0.5536</td>
</tr>
<tr>
<td>LPE</td>
<td>0.4167</td>
<td>1.0000 0.0833 0.0278 0.4444 0.8056 0.2222 0 0.5278 0.6111 0.5000</td>
</tr>
<tr>
<td>HTR</td>
<td>0.4167</td>
<td>0 0.5263 0.5921 0.3026 0 0.7895 0.9474 0.8026 1.0000 0.7237</td>
</tr>
<tr>
<td>QT</td>
<td></td>
<td></td>
</tr>
<tr>
<td>QL</td>
<td></td>
<td></td>
</tr>
<tr>
<td>FLT</td>
<td>0.7500</td>
<td>0.0833 0.2500 0.9170 0.9170 0.0833 0.5833 0.2500 0.0833 0.7500 0.2500</td>
</tr>
<tr>
<td>DBT</td>
<td>0.7500</td>
<td>0.5833 0.0833 0.5833 0.0833 0.4167 0.0833 0.2500 0.7500 0.0833 0.4167</td>
</tr>
<tr>
<td>MTT</td>
<td>0.4167</td>
<td>0.4167 0.5833 0.0833 0.5833 0.2500 0.5833 0.0833 0.917 0.4167 0.0833</td>
</tr>
</tbody>
</table>

\( \bar{T}_i \) | 0.5145 0.5157 0.6168 0.5240 0.3691 0.6054 0.2710 0.5394 0.5819 0.4679 |

Table 7: Aggregated ranking values \( \bar{T}_i \) of packaging material \( i \) for all components

<table>
<thead>
<tr>
<th>Component</th>
<th>Packaging material ( i )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.5145 0.5157 0.6168 0.5240 0.3691 0.6054 0.2710 0.5394 0.5819 0.4679</td>
</tr>
<tr>
<td>2</td>
<td>0.7629 0.7044 0.5255 0.5571 0.4566 0.7028 0.2198 0.5509 0.4766 0.5566</td>
</tr>
<tr>
<td>3</td>
<td>0.7926 0.5601 0.5835 0.5625 0.6714 0.6681 0.5685 0.5069 0.2299 0.5391</td>
</tr>
<tr>
<td>4</td>
<td>0.7607 0.5965 0.6406 0.5613 0.6418 0.6256 0.6111 0.7585 0.722 0.4474</td>
</tr>
<tr>
<td>5</td>
<td>0.6312 0.5841 0.6437 0.5042 0.4792 0.7436 0.5489 0.6307 0.5886 0.5243</td>
</tr>
<tr>
<td>6</td>
<td>0.5631 0.265 0.473 0.5517 0.4618 0.4844 0.4734 0.6107 0.4818 0.3509</td>
</tr>
<tr>
<td>7</td>
<td>0.6893 0.5308 0.6126 0.5222 0.5182 0.5768 0.5539 0.5893 0.4619 0.4248</td>
</tr>
<tr>
<td>8</td>
<td>0.5955 0.5688 0.7042 0.6677 0.4414 0.7064 0.5905 0.6225 0.519 0.6116</td>
</tr>
<tr>
<td>9</td>
<td>0.6363 0.1896 0.5327 0.4706 0.4901 0.7569 0.6742 0.4711 0.5861 0.3902</td>
</tr>
</tbody>
</table>

The system parameters of GA for numerical example are summarized in Table 8. The solution search by GA is implemented using an optimization software package Evolver 4.0 (Palisade Corp. 2004). As the limit of 500 generations is reached, the solution of numerical example is obtained within a negligible execution time. In this typical run, the solution of (3, 1, 1, 1, 6, 8, 1, 3, 1) is found at the 367th generation with the best fitness value of 6.3171 (Figure 6). According to the result, it can be interpreted that packaging material type 3 is selected for component 1 and 8; packaging material type 1 is picked up for component 2, 3, 4, 7 and 9; packaging material type 6 is suggested for component 5, and packaging material type 8 is assigned for component 6.

However, it is important to note that the best fitness obtained may not appear to be the maximum value. In order to provide a comparison, the same problem can be solved with an exhaustive search (ES) approach. In this case, the maximum fitness value of the problem considered is 6.44, and the corresponding chromosome structure is (3, 1, 1, 1, 6, 8, 1, 6, 6). The near optimal solution is a consequence of the fact that the GA search is terminated after the maximum number of 500 generations. For a small-size problem such as the current numerical example, a GA can result in the optimal solution if it is run until no further improvement is obtained. Generally, in the final population, the solution that has the highest fitness value will be chosen as the final design. The designers might run the solution search many times.
Each run will provide a “best” solution that satisfied all the criteria. However, the solutions from each run can be different. Designers can choose a final design by selecting or modifying their results to meet their different application criteria.

Figure 5.7 displays plots of the best fitness values found for 20 runs. Table 9 lists the GA results for a total of 20 runs.

**Table 8: System parameters of GA for numerical example**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Population size (nchr)</td>
<td>50</td>
</tr>
<tr>
<td>Length of chromosome (chrl)</td>
<td>9</td>
</tr>
<tr>
<td>Maximum number of generations</td>
<td>500</td>
</tr>
<tr>
<td>Parent selection procedure</td>
<td>Two chromosomes with highest fitness are selected</td>
</tr>
<tr>
<td>Probability and method for crossover operation</td>
<td>0.6 and single-point crossover</td>
</tr>
<tr>
<td>Probability of mutation operation</td>
<td>0.05</td>
</tr>
<tr>
<td>Regeneration method</td>
<td>Elitism</td>
</tr>
<tr>
<td>Fitness evaluation</td>
<td>Total ranking value</td>
</tr>
<tr>
<td>Termination conditions</td>
<td>When the maximum number of generations is exceeded</td>
</tr>
</tbody>
</table>

**Figure 6: Plots of the best and average fitness values**
Figure 7: Plots of the best fitness values in 20 runs with 500 generations in each run

Table 9: GA results for a total of 20 runs

<table>
<thead>
<tr>
<th>Run</th>
<th>Chromosome structure</th>
<th>Best score</th>
<th>Number of generation that the best fitness was found</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3, 1, 1, 1, 6, 8, 1, 3, 1</td>
<td>6.3171</td>
<td>423</td>
</tr>
<tr>
<td>2</td>
<td>6, 1, 1, 8, 6, 9, 1, 3, 6</td>
<td>6.2952</td>
<td>417</td>
</tr>
<tr>
<td>3</td>
<td>6, 1, 1, 8, 6, 1, 1, 3, 6</td>
<td>6.3765</td>
<td>426</td>
</tr>
<tr>
<td>4</td>
<td>9, 1, 1, 8, 6, 8, 1, 3, 6</td>
<td>6.4006</td>
<td>409</td>
</tr>
<tr>
<td>5</td>
<td>3, 6, 1, 8, 6, 4, 3, 6, 6</td>
<td>6.2419</td>
<td>476</td>
</tr>
<tr>
<td>6</td>
<td>3, 2, 1, 1, 8, 6, 8, 6, 6</td>
<td>6.2814</td>
<td>475</td>
</tr>
<tr>
<td>7</td>
<td>6, 2, 1, 1, 6, 8, 1, 6, 7</td>
<td>6.2873</td>
<td>459</td>
</tr>
<tr>
<td>8</td>
<td>6, 1, 1, 1, 6, 8, 3, 6, 6</td>
<td>6.3518</td>
<td>351</td>
</tr>
<tr>
<td>9</td>
<td>3, 1, 1, 1, 6, 6, 1, 6, 6</td>
<td>6.3136</td>
<td>451</td>
</tr>
<tr>
<td>10</td>
<td>9, 2, 1, 8, 6, 1, 1, 4, 6</td>
<td>6.2580</td>
<td>415</td>
</tr>
<tr>
<td>11</td>
<td>6, 1, 1, 1, 8, 1, 3, 6, 6</td>
<td>6.4263</td>
<td>460</td>
</tr>
<tr>
<td>12</td>
<td>3, 1, 1, 1, 6, 1, 1, 4, 7</td>
<td>6.2709</td>
<td>419</td>
</tr>
<tr>
<td>13</td>
<td>6, 1, 1, 1, 6, 4, 1, 6, 6</td>
<td>6.3695</td>
<td>395</td>
</tr>
<tr>
<td>14</td>
<td>6, 1, 1, 1, 6, 1, 1, 10, 6</td>
<td>6.2861</td>
<td>474</td>
</tr>
<tr>
<td>15</td>
<td>9, 6, 5, 8, 6, 8, 1, 6, 6</td>
<td>6.2215</td>
<td>434</td>
</tr>
<tr>
<td>16</td>
<td>3, 2, 1, 1, 3, 8, 1, 6, 6</td>
<td>6.2815</td>
<td>451</td>
</tr>
<tr>
<td>17</td>
<td>9, 6, 1, 1, 6, 8, 1, 6, 6</td>
<td>6.3449</td>
<td>413</td>
</tr>
<tr>
<td>18</td>
<td>9, 1, 1, 1, 6, 8, 1, 3, 6</td>
<td>6.4028</td>
<td>415</td>
</tr>
<tr>
<td>19</td>
<td>6, 1, 6, 1, 6, 8, 1, 8, 7</td>
<td>6.1374</td>
<td>478</td>
</tr>
<tr>
<td>20</td>
<td>3, 2, 1, 1, 6, 8, 1, 3, 6</td>
<td>6.3792</td>
<td>369</td>
</tr>
</tbody>
</table>

Standard deviation = 0.0709
6. CONCLUSIONS

The combinatorial approach was presented for the packaging material selection (PMS) problem. Both quantitative and qualitative decision criteria were taken into considerations. However, with the help of fuzzy set theory, qualitative criteria evaluated by linguistic expressions were transformed into numerical weights. Weigh scores that resulted from the manipulations in quantitative and qualitative criteria were then used for the GA’s fitness function in searching for optimal or near-optimal design solutions. For the numerical example considered, many near-optimal solutions were found quickly during the early stages of the searching process. As the searching process continued, GA improved further upon the best design was found. An advantage of the method is the flexibility of the genetic algorithm for adapting to different searching needs. The fitness function can be modified with respect to different criteria in order to drive the searching process toward different desired goals.

REFERENCES


